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ODD VERTEX EQUITABLE EVEN LABELING OF QUADRILATERAL SNAKE RELATED GRAPHS

A. LOURDUSAMY[1], J. SHOBANA MARY[2] AND F. PATRICK[3]

[1&3] Department of Mathematics,
St. Xavier's College, Palayamkottai - 627002, India.

[2] M.Sc. Student, Reg. No: 144821ER016,
Department of Mathematics
St. Antony's College of Arts and Science for Women,
Thamaraipadi, Dindigul - 624005, India.

[1]Email: lourdusamy15@gmail.com
[2]Email: jsom17@gmail.com
[3]Email: patrick881990@gmail.com

Abstract: Let G be a graph with p vertices and q edges and $A = \{1,3,5,\dots,q\}$ if q is odd or $A = \{1,3,5,\dots,q+1\}$ if q is even. A graph G is said to be an odd vertex equitable even labeling if there exists a vertex labeling $f: V(G) \to A$ that induces an edge labeling f^* defined by $f^*(uv) = f(u) + f(v)$ for all edges uv such that for all a and b in A, $|v_f(a) - v_f(b)| \leq 1$ and the induced edge labels are $2,4,\dots,2q$, where $v_f(a)$ be the number of vertices v with $f(v) = a$ for $a \in A$. A graph that admits odd vertex equitable even labeling is called an odd vertex equitable even graph. In this paper, we prove that are odd vertex equitable even graphs.

Keywords: vertex equitable labeling, odd vertex equitable even labeling, quadrilateral snake.
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1. INTRODUCTION

All graphs considered here are simple, finite, connected and undirected. Let $G(V,E)$ be a graph with p vertices and q edges. We follow the basic notations and terminologies of graph theory as in [2]. A graph labeling is an assignment of integers to the vertices or edges or both, subject to certain conditions and a detailed survey of graph labeling can be found in [1]. The concept of vertex equitable labeling was introduced by Lourdusamy and Seenivasan [5]. Jeyanthi et al. introduced the concept of odd vertex equitable even labeling in [3].

Definition 1.1: Let G be a graph with p vertices and q edges and $A=\{1,3,5,\dots,q\}$ if q is odd or $A=\{1,3,5,\dots,q+1\}$ if q is even. A graph G is said to be an odd vertex equitable even labeling if there exists a vertex labeling $f: V(G)\to A$ that induces an edge labeling f^* defined by $f^*(uv)=f(u)+f(v)$ for all edges uv such that for all a and b in A, $|v_f(a)-v_f(b)|\le 1$ and the induced edge labels are $2,4,\dots,2q$, where $v_f(a)$ be the number of vertices v with $f(v)=a$ for $a\in A$. A graph that admits odd vertex equitable even labeling is called an odd vertex equitable even graph.

Definition 1.2: The *subdivision of graph* $S(G)$ is obtained from G by subdividing each edge of G with a vertex.

Definition 1.3: The *corona* $G_1 \odot G_2$ of two graphs $G_1(p_1, q_1)$ and $G_2(p_2, q_2)$ is defined as the graph obtained by taking one copy of G_1 and p_1 copies of G_2 and joining the i^{th} vertex of $G_1$with an edge to every vertex in the i^{th} copy of G_2.

Definition 1.4: A *quadrilateral snake* Q_n is obtained from a path $u_1, u_2, u_3, \dots, u_n$ by joining u_i, u_{i+1} to new vertices v_i, w_i respectively and then joining v_i and w_i. That is every edge of the path is replaced by a cycle C_4.

Definition 1.5: A *double quadrilateral snake* $D(Q_n)$ consists of two quadrilateral snakes that have a common path.

Definition 1.6: A *double alternate quadrilateral snake* $DA(Q_n)$ consists of two alternate quadrilateral snakes that have a common path. That is, a double alternate quadrilateral snake is obtained from a path $u_1, u_2, u_3, \dots, u_n$ by joining $u_i$and u_{i+1} to the new vertices v_i, x_i and w_i, y_i respectively and adding the edges $v_i w_i$ and $x_i y_i$.

2. Main Result

Theorem 2.1 The graph $Q_n \odot K_1$ is an odd vertex equitable even graph.

Proof. Let $u_1, u_2, \dots, u_n$ be the vertices of the path P_n. Let $V(Q_n \odot K_1) = \{u_i, u_i^{'} : 1 \le i \le n\} \cup \{v_i, w_i, v_i^{'}, w_i^{'} : 1 \le i \le n-1\}$ and $E(Q_n \odot K_1) = \{u_i u_i^{'} : 1 \le i \le n\} \cup$ $\{u_i u_{i+1}, u_i v_i, v_i w_i, w_i u_{i+1}, v_i v_i^{'}, w_i w_i^{'} : 1 \le i \le n-1\}$. Then, $Q_n \odot K_1$ is of order $6n-4$ and size $7n-6$.

Case (i): n is odd.

Define $f: V(Q_n \odot K_1) \to A = \{1, 3, \dots, 7n-6\}$ as follows:

$$f(u_i) = \begin{cases} 7i-6 & if \;\; i \; is \; odd \; and \; 1 \le i \le n \\ 7i-5 & if \;\; i \; is \; even \; and \; 1 \le i \le n; \end{cases}$$

$$f(u_i^{'}) = \begin{cases} 7i-6 & if \;\; i \; is \; odd \; and \; 1 \le i \le n-1 \\ 7i-7 & if \;\; i \; is \; even \; and \; 1 \le i \le n-1; \end{cases}$$

$$f(v_i) = \begin{cases} 7i-4 & if \;\; i \; is \; odd \; and \; 1 \le i \le n-1 \\ 7i-5 & if \;\; i \; is \; even \; and \; 1 \le i \le n-1; \end{cases}$$

$$f(v_i^{'}) = \begin{cases} 7i-4 & if \;\; i \; is \; odd \; and \; 1 \le i \le n-1 \\ 7i-3 & if \;\; i \; is \; even \; and \; 1 \le i \le n-1; \end{cases}$$

$$f(w_i) = \begin{cases} 7i-2 & if \;\; i \; is \; odd \; and \; 1 \le i \le n-1 \\ 7i-1 & if \;\; i \; is \; even \; and \; 1 \le i \le n-1; \end{cases}$$

$$f(w_i^{'}) = \begin{cases} 7i & if \;\; i \; is \; odd \; and \; 1 \le i \le n-1 \\ 7i-1 & if \;\; i \; is \; even \; and \; 1 \le i \le n-1. \end{cases}$$

It can be verified that the induced edge labels of $Q_n \odot K_1$ are $2, 4, \ldots, 14n-12$. Hence the graph $Q_n \odot K_1$ is an odd vertex equitable even graph.

Case (ii): n is even.

We define a vertex labeling $f: V(Q_n \odot K_1) \rightarrow A = \{1, 3, \ldots, 7n-5\}$ as follows. Assign the labels to the vertices $u_i, u_i^{'}$ for $1 \le i \le n$ and $v_i, w_i, v_i^{'}, w_i^{'}$ for $1 \le i \le n-1$ as in Case (i). It can be verified that the induced edge labels of $Q_n \odot K_1$ are $2, 4, \ldots, 14n-12$. Hence the graph $Q_n \odot K_1$ is an odd vertex equitable even graph.

Example 2.2: An odd vertex equitable even labeling of $Q_5 \odot K_1$ is shown in the Figure 2.1.

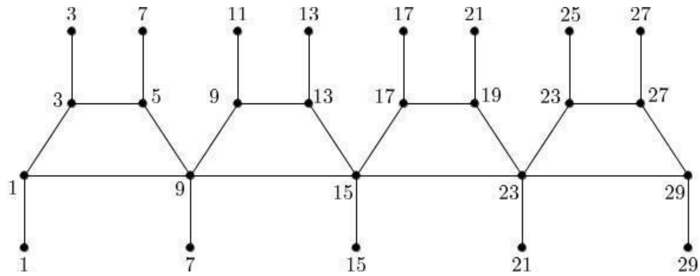

Figure 2.1

Theorem 2.3 The graph $D(Q_n)$ is an odd vertex equitable even graph.

Proof. The quadrilateral snake is obtained from a path $u_1, u_2, \dots, u_n$. Let $V(D(Q_n)) = \{v_i, w_i, v_i', w_i' : 1 \leq i \leq n-1\} \cup \{u_i : 1 \leq i \leq n\}$ and $E(D(Q_n)) = \{u_i u_{i+1} : 1 \leq i \leq n-1\} \cup \{u_{2i-1}v_i, u_{2i-1}v_i', u_{2i}w_i, u_{2i}w_i', v_i w_i, v_i' w_i' : 1 \leq i \leq n-1\}$.

Then, $D(Q_n)$ is of order $5n-4$ and size $7n-7$.

Case (i): n is odd.

Define $f: V(D(Q_n)) \to A = \{1,3,\dots,7n-6\}$ as follows:

$$f(u_i) = \begin{cases} 7i-6 & if \;\; i \; is \; odd \; and \; 1 \leq i \leq n \\ 7i-7 & if \;\; i \; is \; even \; and \; 1 \leq i \leq n; \end{cases}$$

$$f(v_i) = \begin{cases} 7i-6 & if \;\; i \; is \; odd \; and \; 1 \leq i \leq n-1 \\ 7i-5 & if \;\; i \; is \; even \; and \; 1 \leq i \leq n-1; \end{cases}$$

$$f(w_i) = \begin{cases} 7i-2 & if \;\; i \; is \; odd \; and \; 1 \leq i \leq n-1 \\ 7i-3 & if \;\; i \; is \; even \; and \; 1 \leq i \leq n-1; \end{cases}$$

$$f(v_i') = \begin{cases} 7i-4 & if \;\; i \; is \; odd \; and \; 1 \leq i \leq n-1 \\ 7i-3 & if \;\; i \; is \; even \; and \; 1 \leq i \leq n-1; \end{cases}$$

$$f(w_i') = \begin{cases} 7i & if \;\; i \; is \; odd \; and \; 1 \leq i \leq n-1 \\ 7i-1 & if \;\; i \; is \; even \; and \; 1 \leq i \leq n-1. \end{cases}$$

It can be easily verified that the induced edge labels of $D(Q_n)$ are $2,4,\dots,14n-14$. Hence the graph $D(Q_n)$ is an odd vertex equitable even graph.

Case (ii): n is even.

We define a vertex labeling $f: V(D(Q_n)) \to A = \{1,3,\dots,7n-7\}$ as follows. Assign the labels to the vertices u_i for $1 \le i \le n$ and $v_i, w_i, v_i^{'}, w_i^{'}$ for $1 \le i \le n-1$ as in Case (i). It can be verified that the induced edge labels of $D(Q_n)$ are $2,4,\dots,14n-14$. Hence the graph $D(Q_n)$ is an odd vertex equitable even graph.

Example 2.4: An odd vertex equitable even labeling of $D(Q_4)$ is shown in the Figure 2.2.

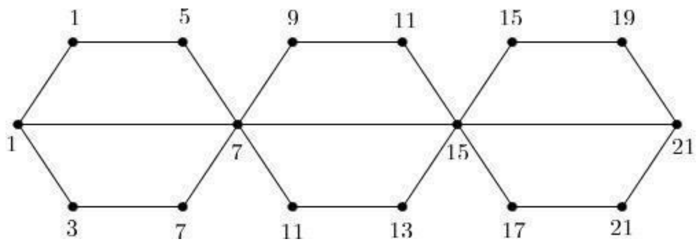

Figure 2.2

Theorem 2.5 The graph $DA(Q_n)$ is an odd vertex equitable even graph.

Proof. The quadrilateral snake is obtained from a path $u_1, u_2, \dots, u_n$. Let $V(DA(Q_n)) = \{v_i, w_i, v_i^{'}, w_i^{'} : 1 \le i \le n-1\} \cup \{u_i : 1 \le i \le n\}$ and $E(DA(Q_n)) =$
$\{u_i u_{i+1} : 1 \le i \le n-1\} \cup \left\{u_i v_i, u_i v_i^{'}, u_{i+1} w_i, u_{i+1} w_i^{'}, v_i w_i, v_i^{'} w_i^{'} : 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor\right\}$.

Then,

$$V(DA(Q_n)) = \begin{cases} 3n-2 & if\ \ n\ \ is\ odd \\ 3n & if\ \ n\ is\ even \end{cases}$$

$$E(DA(Q_n)) = \begin{cases} 4n-4 & if\ \ n\ \ is\ odd \\ 4n-1 & if\ \ n\ is\ even \end{cases}$$

Case (i): n is odd.

Define $f: V(DA(Q_n)) \longrightarrow A = \{1,3, \ldots, 4n-3\}$ as follows:

$$f(u_i) = \begin{cases} 4i-3 & if\ \ i\ is\ odd\ and\ 1 \le i \le n \\ 4i-1 & if\ \ i\ is\ even\ and\ 1 \le i \le n; \end{cases}$$

$$f(v_i) = 8i-7, \quad 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor;$$

$$f(w_i) = 8i-3, \quad 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor;$$

$$f(v_i^{'}) = 8i-5, \quad 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor;$$

$$f(w_i^{'}) = 8i-1, \quad 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor.$$

It can be easily verified that the induced edge labels of $DA(Q_n)$ are $2,4, \ldots, 8n-8$. Hence the graph $DA(Q_n)$ is an odd vertex equitable even graph.

Case (ii): n is even.

We define a vertex labeling $f: V(DA(Q_n)) \longrightarrow A = \{1,3, \ldots, 4n-1\}$ as follows. Assign the labels to the vertices u_i for $1 \le i \le n$ and $v_i, w_i, v_i^{'}, w_i^{'}$ for $1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor$ as in Case (i). It can be verified that the induced edge labels of $DA(Q_n)$ are $2,4, \ldots, 8n-2$. Hence the graph $DA(Q_n)$ is an odd vertex equitable even graph.

Example 2.6: An odd vertex equitable even labeling of $DA(Q_5)$ is shown in the Figure 2.3.

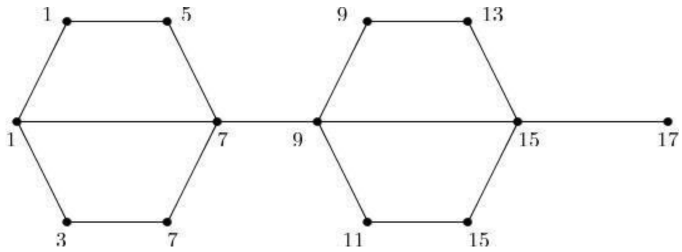

Figure 2.3

Theorem 2.7 The graph $S(Q_n)$ is an odd vertex equitable even graph.

Proof. Let $u_1, u_2, \ldots, u_n$ be the vertices of the path P_n. Let $V(S(Q_n)) = \{u_i : 1 \le i \le n\} \cup \{v_i, w_i, v_i', w_i', u_i', x_i : 1 \le i \le n-1\}$ and $E(S(Q_n)) = \{u_i u_i', u_i' u_{i+1}, u_i v_i', v_i' v_i, v_i x_i,$ $x_i w_i, w_i w_i', w_i' u_{i+1} : 1 \le i \le n-1\}$. Then, $S(Q_n)$ is of order $7n-6$ and size $8n-8$.

Define $f: V(S(Q_n)) \to A = \{1,3,\ldots,8n-7\}$ as follows:

$f(u_i) = 8i-7,\ 1 \le i \le n;$

$f(u_i') = 8i-1,\ 1 \le i \le n-1;$

$f(v_i) = 8i-5,\ 1 \le i \le n-1;$

$f(v_i') = 8i-7,\ 1 \le i \le n-1;$

$f(w_i) = 8i-1,\ 1 \le i \le n-1;$

$f(w_i') = 8i-3,\ 1 \le i \le n-1;$

$f(x_i) = 8i-5,\ 1 \le i \le n-1.$

It can be easily verified that the induced edge labels of $S(Q_n)$ are $2,4,\ldots,16n-16$. Hence the graph $S(Q_n)$ is an odd vertex equitable even graph.

Example 2.8: An odd vertex equitable even labeling of $S(Q_4)$ is shown in the Figure 2.4.

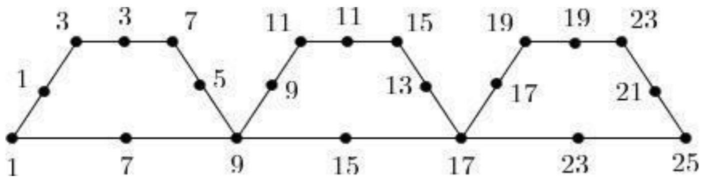

Figure 2.4

Theorem 2.9 The graph $S(D(Q_n))$ is an odd vertex equitable even graph.

Proof. Let $V(S(D(Q_n))) = \{u_i : 1 \leq i \leq n\} \cup \{v_i, w_i, v_i', w_i', u_i', x_i, x_i', z_i, z_i', y_i, y_i' : 1 \leq i \leq n-1\}$ and $E(S(D(Q_n))) = \{u_i u_i', u_i' u_{i+1}, u_i v_i', v_i' v_i, v_i z_i, z_i w_i, w_i w_i', w_i' u_{i+1} : 1 \leq i \leq n-1\} \cup \{u_i x_i', x_i' x_i, x_i z_i', z_i' y_i, y_i y_i', y_i' u_{i+1} : 1 \leq i \leq n-1\}$. Then, $S(D(Q_n))$ is of order $12n - 11$ and size $14n - 14$.

Define $f : V(S(D(Q_n))) \to A = \{1, 3, \ldots, 14n - 13\}$ as follows:

$f(u_i) = 14i - 13,\ 1 \leq i \leq n;$

$f(u_i') = 14i - 1,\ 1 \leq i \leq n-1;$

$f(v_i) = 14i - 11,\ 1 \leq i \leq n-1;$

$f(v_i') = 14i - 13,\ 1 \leq i \leq n-1;$

$f(w_i) = 14i - 7,\ 1 \leq i \leq n-1;$

$f(w_i') = 14i - 9,\ 1 \leq i \leq n-1;$

$f(x_i) = 14i - 5,\ 1 \leq i \leq n-1;$

$f(x_i') = 14i - 7,\ 1 \leq i \leq n-1;$

$f(y_i) = 14i - 1,\ 1 \leq i \leq n-1;$

$f(y_i') = 14i - 3,\ 1 \leq i \leq n-1;$

$f(z_i) = 14i - 11,\ 1 \leq i \leq n-1;$

$f(z_i') = 14i - 5,\ 1 \leq i \leq n-1.$

It can be easily verified that the induced edge labels of $S(D(Q_n))$ are $2,4,\ldots,28n-28$. Hence the graph $S(D(Q_n))$ is an odd vertex equitable even graph.

Example 2.10: An odd vertex equitable even labeling of $S(D(Q_4))$ is shown in the Figure 2.5.

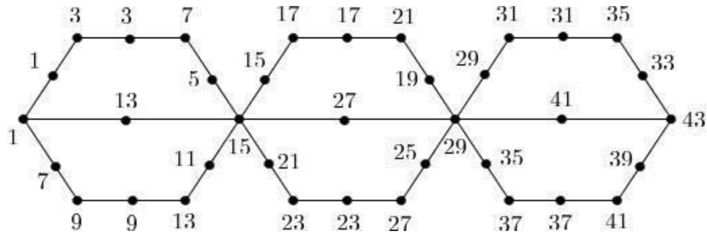

Figure 2.5

Theorem 2.11 The graph $S(DA(Q_n))$ is an odd vertex equitable even graph.

Proof.

Let

$V(S(DA(Q_n))) = \{u_i : 1 \le i \le n\} \cup \{v_i, w_i, v_i^{'}, w_i^{'}, u_i^{'}, x_i, x_i^{'}, z_i, z_i^{'}, y_i, y_i^{'} : 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor\}$ and $E(S(DA(Q_n))) = \{u_i u_i^{'}, u_i^{'} u_{i+1}, : 1 \le i \le n-1\} \cup \{u_i v_i^{'}, v_i^{'} v_i, v_i z_i, z_i w_i, w_i w_i^{'}, w_i^{'} u_{i+1}, u_i x_i^{'}, x_i^{'} x_i, x_i z_i^{'}, z_i^{'} y_i, y_i y_i^{'}, y_i^{'} u_{i+1} : 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor\}$. Then, $S(DA(Q_n))$ is of order $7n-1$ and size $8n-2$.

$$V(DA(Q_n)) = \begin{cases} 7n-6 & if \;\; n \; is\, odd \\ 7n-1 & if \;\; n\, is\, even \end{cases}$$

$$E(DA(Q_n)) = \begin{cases} 8n-8 & if \;\; n \; is\, odd \\ 8n-2 & if \;\; n\, is\, even \end{cases}$$

Case (i): n is odd.

Define $f: V(S(DA(Q_n))) \to A = \{1,3,\ldots,8n-7\}$ as follows:

$$f(u_i) = \begin{cases} 8i - 7 & if \;\; i \text{ is odd and } 1 \le i \le n \\ 8i - 1 & if \;\; i \text{ is even and } 1 \le i \le n; \end{cases}$$

$$f(u_i^{'}) = \begin{cases} 8i + 5 & if \;\; i \text{ is odd and } 1 \le i \le n-1 \\ 8i - 1 & if \;\; i \text{ is even and } 1 \le i \le n-1; \end{cases}$$

$$f(v_i) = 16i - 13, \;\; 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor;$$

$$f(v_i^{'}) = 16i - 15, \;\; 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor;$$

$$f(w_i) = 16i - 9, \;\; 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor;$$

$$f(w_i^{'}) = 16i - 11, \;\; 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor;$$

$$f(x_i) = 16i - 7, \;\; 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor;$$

$$f(x_i^{'}) = 16i - 9, \;\; 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor;$$

$$f(y_i) = 16i - 3, \;\; 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor;$$

$$f(y_i^{'}) = 16i - 5, \;\; 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor;$$

$$f(z_i) = 16i - 13, \;\; 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor;$$

$$f(z_i^{'}) = 16i - 7, \;\; 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor.$$

It can be easily verified that the induced edge labels of $S(DA(Q_n))$ are $2,4, \dots, 16n - 16$. Hence the graph $S(DA(Q_n))$ is an odd vertex equitable even graph.

Case (ii): n is even.

We define a vertex labeling $f: V(S(DA(Q_n))) \to A = \{1,3, \dots, 8n - 1\}$ as follows. Assign the labels to the vertices u_i for $1 \le i \le n$, $u_i^{'}$ for $1 \le i \le n - 1$ and $v_i, w_i, x_i, y_i, z_i, v_i^{'}, w_i^{'}, x_i^{'}, y_i^{'}, z_i^{'}$ for $1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor$ as in Case (i). It can be verified that the induced edge labels of $S(DA(Q_n))$

are $2,4,\ldots,16n-4$. Hence the graph $S(DA(Q_n))$ is an odd vertex equitable even graph.

Example 2.12: An odd vertex equitable even labeling of $S(DA(Q_5))$ is shown in the Figure 2.6.

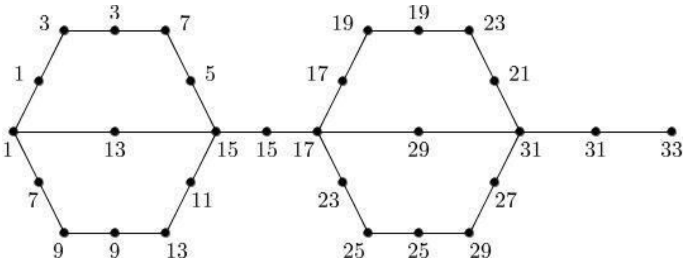

Figure 2.6

Theorem 2.13: The graph $S(P_n \odot K_1)$ is an odd vertex equitable even graph.

Proof: Let $u_1, u_2, u_3, \ldots, u_n$ be the vertices of path P_n. Let $V(S(P_n \odot K_1)) = \{u_i, v_i, v_i' : 1 \le i \le n\} \cup \{u_i' : 1 \le i \le n-1\}$ and $E(S(P_n \odot K_1)) = \{u_i u_i', u_i' u_{i+1} : 1 \le i \le n-1\} \cup \{v_i v_i', u_i v_i' : 1 \le i \le n\}$.

Then $S(P_n \odot K_1)$ is of order $4n-1$ and size $4n-2$.

Define $f: V(S(P_n \odot K_1)) \to A = \{1,3,5,\ldots,4n-1\}$ as follows:

$$f(u_i) = \begin{cases} 4i-1 & if \;\; i \; is \; odd \; and \; 1 \le i \le n \\ 4i-3 & if \;\; i \; is \; even \; and \; 1 \le i \le n; \end{cases}$$

$$f(u_i') = \begin{cases} 4i-1 & if \;\; i \; is \; odd \; and \; 1 \le i \le n-1 \\ 4i-3 & if \;\; i \; is \; even \; and \; 1 \le i \le n-1; \end{cases}$$

$$f(v_i) = \begin{cases} 4i-3 & if \;\; i \; is \; odd \; and \; 1 \le i \le n \\ 4i-1 & if \;\; i \; is \; even \; and \; 1 \le i \le n; \end{cases}$$

$$f(v_i') = \begin{cases} 4i-3 & if \;\; i \; is \; odd \; and \; 1 \le i \le n-1 \\ 4i-1 & if \;\; i \; is \; even \; and \; 1 \le i \le n-1; \end{cases}$$

$f(v_n') = 4i - 3.$

It can be easily verified that the induced edge labels of $S(P_n \odot K_1)$ are $2, 4, \ldots, 8n - 4$. Hence the graph $S(P_n \odot K_1)$ is an odd vertex equitable even graph.

Example 2.14: An odd vertex equitable even labeling of $S(P_4 \odot K_1)$ is shown in the Figure 2.7.

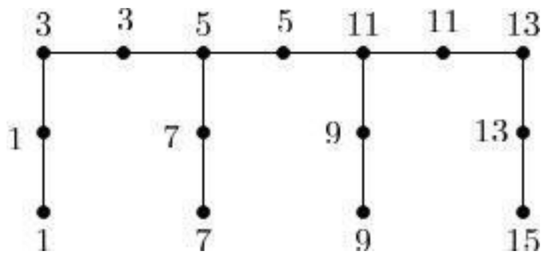

Figure 2.7

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